# Shell-model calculations of neutrino scattering from $^{12}$C

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## Abstract

Neutrino reaction cross-sections, $(\nu_\mu, \mu^-)$, $(\nu_e, e^-)$, $\mu$-capture and photoabsorption rates on $^{12}$C are computed within a large-basis shell-model framework, which included excitations up to $4\hbar\omega$. When ground-state correlations are included with an open $p$-shell the predictions of the calculations are in reasonable agreement with most of the experimental results for these reactions. Woods-Saxon radial wave functions are used, with their asymptotic forms matched to the experimental separation energies for bound states, and matched to a binding energy of 0.01 MeV for unbound states. For comparison purposes, some results are given for harmonic oscillator radial functions. Closest agreement between theory and experiment is achieved with unrestricted shell-model configurations and Woods-Saxon radial functions. We obtain for the neutrino-absorption inclusive cross sections: $\overline{\sigma} = 13.8 \times 10^{-40}$ cm$^2$ for the $(\nu_\mu, \mu^-)$ decay-in-flight flux in agreement with the LSND datum of $(12.4 \pm 1.8) \times 10^{-40}$ cm$^2$; and $\overline{\sigma} = 12.5 \times 10^{-42}$ cm$^2$ for the $(\nu_e, e^-)$ decay-at-rest flux, less than the experimental result of $(14.4 \pm 1.2) \times 10^{-42}$ cm$^2$.

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## I. INTRODUCTION

One of the key neutrino-nucleus reactions measured at the two accelerator based neutrino-oscillations searches, LSND at the LANSCE facility in Los Alamos and KARMEN at the ISIS facility in the U.K., is the scattering of neutrinos from carbon in the liquid scintillation detector. The neutrino source at both experiments comes from the decay of pions produced in the beam stop. The vast majority of pions decay at rest, and the electron neutrinos thus produced have enough energy to cause a nuclear charge-exchange reaction on carbon, $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$. At LSND 3.4% of the pions decay in flight, producing muon neutrinos of sufficient energy to interact via the reaction $^{12}\mathrm{C}(\nu_\mu, \mu^-)^{12}\mathrm{N}$. The signal for the oscillation of decay-in-flight (DIF) muon neutrinos ($\nu_\mu \rightarrow \nu_e$) at LSND is the appearance of high energy electrons from the $\nu_e C \rightarrow Ne^-$ reaction. Extracting oscillation parameters from this search requires knowledge of the expected cross-section. In addition, the measured $\nu_\mu\mathrm{C}{\rightarrow}\mathrm{N}\mu^-$ cross-section acts as a test of the $\nu_\mu$ DIF flux and of the detector efficiency. The KARMEN experiment also has the efficiency to measure the $\bar{\nu}C \rightarrow B\mu^+$ reaction. At both LSND and KARMEN the inclusive $\nu_e\mathrm{C}{\rightarrow}\mathrm{X}e^-$ and the exclusive $\nu_e\mathrm{C}{\rightarrow}^{12}\mathrm{N}_{g.s.}e^-$ cross-sections are measured for the electron neutrinos from the decay of the pion at rest. The flux of the decay-at-rest (DAR) electron neutrinos is given by the Michel spectrum, and the $\nu_e$C cross sections provide a strong constraint on the nuclear-structure models used for carbon.

The inclusive $^{12}\mathrm{C}(\nu_\mu, \mu^-)\mathrm{X}$ cross section for the DIF $\nu_\mu$ flux at LSND has been measured. The first calculations for this cross section were carried out by the the Caltech group [1] using a continuum Random Phase Approximation (RPA) model. The calculated cross sections over-estimated experiment by almost a factor of two, and suggested that the measured cross sections may be inconsistent with other observables for $^{12}$C. The most recent version of this work [2] allows for a partial occupancy of the nuclear subshells in the continuum RPA, a feature that brings theory and experiment closer together, but still does not remove the discrepancy fully. The other key observables that need to be considered are $\mu$-capture, the DAR $^{12}\mathrm{C}(\nu_e, e^-)X$, $(e, e')$, photo-absorption, and $\beta$-decay. These different probes involve different energy and momentum transfers and, thus, constrain different aspects of the calculations. The disagreement between theory and experiment for the DIF $\nu_\mu$C cross section prompted new calculations [3,4] in an effort to uncover possible shortcomings of the RPA calculations. In the present work we perform a series of shell-model based calculations, which include excitations up to $4\hbar\omega$ for $^{12}$C, $^{12}$N and $^{12}$B, each involving different assumptions and approximations.

The DAR neutrino spectrum involves neutrino energies $0 - 52$ MeV, with an average neutrino energy $E_\nu \sim 32$ MeV. The $Q$-value for the charge-exchange reaction $(\nu_e, e^-)$ on $^{12}$C is about 17 MeV. The DAR inclusive cross-section is then dominated by low multipoles ($1^+$, $1^-$, $2^-$) and by excitation of the giant dipole resonances. In the case of the $\nu_\mu$C cross section, the DIF muon neutrino flux involves an average neutrino energy of about $E_\nu \sim 150$ MeV, but the flux is finite up to $E_\nu \sim 250$ MeV. The $Q$-value for the $(\nu_\mu, \mu^-)$ reaction is close to 123 MeV. Calculations for this cross section need to include both a good description of the giant resonance region ($E_x \sim 15 - 40$ MeV of excitation in $^{12}$C) and of higher excitation energy regions (up to $80 - 100$ MeV). Furthermore, all multipoles $\lambda \sim 0 - 5$ make significant contributions to the inclusive cross section.

## II. GENERAL NUCLEAR STRUCTURE CONSIDERATIONS

In the simplest shell model, $^{12}$C consists of four neutrons and four protons in the $p$-shell outside a closed $^4$He core. Excited states reached by the $(\nu, \ell^-)$ reactions are simple particle-hole states built on the ground state. However, the structure of both the $^{12}C_{g.s.}$ and the continuum states of $^{12}$N also involve more sophisticated configurations and configuration mixing. There is a limit on the size of the model space that can be included in any calculation, so that calculated cross sections necessarily involve some level of approximation. In the case of the neutrino reactions on carbon the approximations that are most likely to affect the predicted cross sections are ($a$) the $^{12}$C ground state $p$-shell structure, ($b$) the treatment of ground state correlations beyond the $p$-shell, ($c$) the model space truncation, especially for the final states, ($d$) configuration mixing in the final states, and ($e$) the nuclear radial wave functions. In this paper we examine the effect of each of these on the predicted cross section.

An approximation that is inherent in the continuum RPA calculations of Kolbe *et al.* [1] is the restriction of the $^{12}$C ground state to a closed $0p_{3/2}$ shell. This approximation is necessary to build a spectrum of particle-hole states representing the excitations in $^{12}$N and to evaluate their transition cross sections in the RPA. That this approximation is very poor for the lowest positive-parity states of configuration $(p_{3/2}^{-1}, p_{1/2})$ is well known and recognized in the calculations of Kolbe *et al.* [1], who reduce their calculated cross section to the $^{12}N_{g.s.}$ by a factor of four. A smaller reduction factor of order 1.5 is obtained when this work is extended [2] to include partial occupancies for the $p_{1/2}$ subshell. An additional reduction of the cross section to the other low-lying positive parity states, particularly to the $2^+$ state at 0.96 MeV of excitation in $^{12}$N, should also be included. It is unclear whether any further correction is required for particle-hole states of energy $1\hbar\omega$, $2\hbar\omega$ ... above these lowest positive-parity states in $^{12}$N. Kolbe *et al.* have argued against additional suppression factors, other than the one that has been applied to the GT transition to the $^{12}$N ground state.

In the case of particle-hole excitations out of the $p$-shell, the $p$-shell structure of the ground state does not affect the total sum-rule for a given operator, which is determined by the number of particles in the $p$-shell. The total spin independent $\Delta S = 0$ multipole strength is independent of the $p$-shell structure of the ground state. However, the total spin-dependent $\Delta S = 1$ strength is distributed differently over the different spin-multipoles for different $p$-shell ground states [5,6]. Thus, for flux-averaged neutrino reactions the predicted cross section will, in general, be model dependent, reflecting the model ground state spin structure.

A reasonable approximation for the structure of $^{12}$C is the $p$-shell equivalent of three $\alpha$-particles. This corresponds to an $L = 0$ $S = 0$ ground state, with good SU(4) symmetry, $[\widetilde{444}]$. The Cohen-Kurath interaction [7] predicts that this state makes up 78% of the $^{12}C_{g.s.}$ wave function. In contrast, the closed $p_{3/2}$-state contains 16% $S = 0$ and 6%$[\widetilde{444}]$ symmetry, Table I. Thus, the spin response for the closed $p_{3/2}$ state will differ from that of the Cohen-Kurath ground state. Determining the degree to which this model dependence is reflected in the predicted neutrino cross sections is a main aim of the present work.

Proton threshold in $^{12}$N is at 0.601 MeV, so that all levels but the ground state are unstable. States up to about 8 MeV of excitation in $^{12}$N have been studied via the $^{12}C(p, n)$ and $(n, p)$ charge exchange reactions. The structure of these states can be understood largely

within a (0+1)$\hbar\omega$ shell-model calculation, although the predictions of these Tamm-Dancoff calculations overestimate the charge exchange cross sections by about a factor of two. Little detailed information exists for higher energy regions of the $^{12}$N continuum.

The unbound nature of the states in $^{12}$N is in strong contrast with the deeply bound $p$-shell particles in the $^{12}$C ground state. Thus, there will be a strong lack of overlap between the initial- and final-state radial functions. The continuum RPA calculations of Kolbe *et al.* [1] explicitly treat the continuum nature of the excited states of $^{12}$N. In contrast, the present shell model calculations are discrete state calculations. Calculations of the $^{12}$C$(p,n)$ and $^{12}$C$(n,p)$ reactions which took binding energy effects into account via the use of Woods-Saxon single-particle wave functions found a $\sim$30% suppression of the predicted cross sections over the predictions using harmonic oscillator wave functions. In the present work we believe it is essential to work with Woods-Saxon radial functions, but for comparison purposes we will give some results with harmonic oscillator single-particle functions.

## III. SHELL-MODEL CALCULATIONS

### A. Model Space

To investigate the effect of the approximations inherent in any model calculations of the inclusive neutrino cross sections on carbon we mount a series of shell-model calculations, and present calculations in four separate model spaces. The spaces include excitations up to 4$\hbar\omega$, and we label these model spaces (1) Closed-shell TDA, (2) Closed-shell RPA, (3) Closed-shell RPA +2p-2h, and (4) Unrestricted shell model. The labels represent the model spaces:

1. *Closed-shell TDA*. The $^{12}$C ground state is described as a closed $0p_{3/2}$ shell and the $^{12}$N excitations as one particle-one hole (1p-1h) states:

$$\begin{aligned} |^{12}C\rangle &= |\ 0\rangle, \\ |^{12}N\rangle &= |\ (h^{-1},p)^{n\hbar\omega} J,T\rangle \qquad n=0,1,2,3,4 \end{aligned} \tag{1}$$

The 1p-1h states are labeled by their energy of excitation, $n\hbar\omega$, in an oscillator model. For example, states $(p_{3/2}^{-1},p_{1/2})$ are $0\hbar\omega$ excitations, $(s_{1/2}^{-1},p_{1/2})$ and $(p_{3/2}^{-1},sd)$ are $1\hbar\omega$ excitations, $(s_{1/2}^{-1},sd)$ and $(p_{3/2}^{-1},pf)$ are $2\hbar\omega$ excitations, and so on. We present calculations up to $4\hbar\omega$ excitation. A shell-model calculation in this model space is equivalent to the Tamm-Dancoff Approximation (TDA) for particle-hole excitations.

2. *Closed-shell RPA*. In this case, 2p-2h excitations are included in the $^{12}$C ground-state wave function

$$\begin{aligned} |^{12}C\rangle &= |\ 0\rangle + |\ (h_1^{-1},h_2^{-1})J_1,T_1;(p_1,p_2)J_1,T_1:00\rangle \\ |^{12}N\rangle &= |\ (h^{-1},p)^{n\hbar\omega} J,T\rangle \qquad n=0,1,2,3,4 \end{aligned} \tag{2}$$

Here $h_1,h_2$ span the hole orbitals, $0s_{1/2}$ and $0p_{3/2}$, while $p_1,p_2$ span particle orbitals $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, .... The highest-energy orbital included in the particle space matches the highest-energy orbital, $p$, in the 1p-1h basis of energy $n\hbar\omega$. Although we have labeled this calculation, RPA, there are more 2p-2h states included in the $^{12}$C wave function here than are

normally present in an RPA calculation. This is because the 2p-2h correlations introduced in RPA are restricted to the type

$$\mid (h_1^{-1}, p_1)^{n\hbar\omega} J, T; (h_2^{-1}, p_2)^{n\hbar\omega} J, T : 00\rangle,$$

where the 2p-2h states are made up only from the coupling of two 1p-1h states of spin, isospin $J, T$ that comprise the basis states of $^{12}$N. Not only is this 2p-2h basis smaller than that in Eq. (2) it also is not fully antisymmetrized. Hole states $h_1$ and $h_2$, and particle states $p_1$ and $p_2$ are not antisymmetrized with respect to each other. This shortcoming is not present in the basis of Eq. (2), where the coupling order shown makes it easy to antisymmetrize the states. Although, as we have just explained, a calculation in this model space is more than just RPA, this case is the closest we have to the calculations of Kolbe *et al.* [1].

3. *Closed-shell RPA + 2p-2h.* In this case, we add some 2p-2h configurations to the 1p-1h basis states of $^{12}$N:

$$\begin{aligned} |^{12}C\rangle &= \mid 0\rangle + \mid (h_1^{-1}, h_2^{-1}) J_1, T_1; (p_1, p_2) J_1, T_1 : 00\rangle \\ |^{12}N\rangle &= \mid (h^{-1}, p)^{n\hbar\omega} J, T\rangle + \mid (p_{3/2}^{-1}, h^{-1}) J_3, T_3; (p_{1/2}, p) J_4, T_4 : J, T\rangle \qquad n = 0, 1, 2, 3, 4 \end{aligned} \quad (3)$$

Note that among the 2p-2h configurations for $^{12}$N, one of the holes is restricted to the $0p_{3/2}$ orbital and one of the particles to the $0p_{1/2}$ orbital. In this way, we are moving further away from the assumption that the $^{12}$C wave function is a closed $p_{3/2}$ shell. Again the other hole orbital, $h$, may span the hole orbitals, $0s_{1/2}$ and $0p_{3/2}$, while the other particle orbit, $p$, spans orbitals $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, ... up to the same maximum characterizing the 1p-1h states.

4. *Fully Unrestricted Shell Model.* Finally, we move to an unrestricted shell-model calculation, with full mixing between all configurations:

$$\begin{aligned} |^{12}C\ g.s.> &= \mid (p_{3/2}p_{1/2})^8 : 00> + \mid (p_{3/2}p_{1/2})^6 (1s0d)^2 : 00> \\ &\quad + \mid (p_{3/2}p_{1/2})^7 (1p0f) : 00> + \mid (0s)^{-1} (p_{3/2}p_{1/2})^8 (1s0d) : 00> \\ \\ |^{12}N\ \ \pi+> &= \mid (p_{3/2}p_{1/2})^8 : JT> + \mid (p_{3/2}p_{1/2})^6 (1s0d)^2 : JT> \\ &\quad + \mid (p_{3/2}p_{1/2})^7 (1p0f) : JT> + \mid (0s)^{-1} (p_{3/2}p_{1/2})^8 (1s0d) : JT> \\ \\ |^{12}N\ \ \pi-> &= \mid (p_{3/2}p_{1/2})^7 (1s0d) : JT> + \mid (0s)^{-1} (p_{3/2}p_{1/2})^9 : JT> . \end{aligned} \quad (4)$$

Spurious center-of-mass states were eliminated exactly by making a transformation to an orthonormal basis for which the expectation value of the centre-of-mass Hamiltonian is zero, $< H_{cm} >= 0$. Our $2\hbar\omega$ shell model basis contains $\sim 2000$ states for each multipole, which is reduced by $\sim 25$ when transformed to a non-spurious basis. Because of computational limitations we are only able to take our "unrestricted" shell model calculations up to $2\hbar\omega$. However, we assume an extrapolation of a $2\hbar\omega$ shell-model calculation to $4\hbar\omega$ scales in the same way as the closed-shell RPA + 2p-2h calculation.

### B. Effective Interactions

Having selected the appropriate model spaces, we must specify the operative effective interactions to use in these spaces. We choose interactions that have been fitted to reproduce experimental spectra, namely: For interactions among the $p$-shell orbitals, we use the

interaction of Cohen and Kurath [7], (8-16)2BME; for interactions among the $s,d$-orbitals, we use the universal $s,d$-interaction (USD) of Wildenthal [8]; for interactions among the $p,f$-shell orbitals, we use the $G$-matrix of Kuo and Brown [9] as modified by Zuker [10] and known as KB3. All other two-body matrix elements, including the cross-shell interactions, were calculated using the Millener-Kurath [11] parameterization in terms of central, spin-orbit, and tensor interactions, whose radial forms are Yukawa functions of various strengths and ranges in different spin-isospin channels. The parameters are determined in fits to spectra of unnatural parity states in $p$-shell nuclei, and so are particularly appropriate for the cross-shell interactions between the $p$- and $s,d$-shell shells. We, however, use this interaction for all cross-shell matrix elements required.

Finally, the single-particle energies must be specified. For the (8-16)2BME, USD and KB3 effective interactions these energies are given as part of the fitted interaction, but are referenced to their respective cores in their shell-model usage, namely $^4$He, $^{16}$O and $^{40}$Ca. In the present work, we wish to make our reference core a closed $p_{3/2}$ shell at $^{12}$C, so these single-particle energies are shifted according to the formula

$$\epsilon_j^A = \epsilon_j^B + \sum_{h,J,T} \frac{(2J+1)(2T+1)}{2(2j+1)} \langle (j,h)J,T \mid V \mid (j,h)J,T \rangle, \tag{5}$$

where $\epsilon_j^A$ is the single-particle energy of an orbital, $j$, relative to a core $A$, $\epsilon_j^B$ relative to a core $B$, $A > B$, and the sum, $h$, is over all the states occupied in $A$ but unoccupied in $B$. Since only the relative separation of single-particle energies is relevant in a shell-model calculation we set the single-particle energy of the $0p_{1/2}$ orbital at its experimental value of $-4.95$ MeV. For orbitals above the $p,f$-shell, we have little guidance in choosing the single-particle energies. We use, therefore, a formula given in Bohr-Mottelson [12]

$$\epsilon_{n,l,j} = const. + v_{ls}\hbar\omega(\mathbf{l}.\mathbf{s}) + v_{ll}\hbar\omega\left(\mathbf{l}^2 - \langle \mathbf{l}^2 \rangle_N\right), \tag{6}$$

$$\langle \mathbf{l}^2 \rangle_N = \frac{1}{2}N(N+3), \tag{7}$$

where $N = 2n + l$ is the principal quantum number for the oscillator orbital, $n$ the number of radial nodes (excluding the origin and infinity) and $l$ the orbital angular momentum quantum number. We use the values $v_{ls} = -0.127$ and $v_{ll} = -0.03$ from [12]. The choice of $\hbar\omega$ to be used in this formula should be one appropriate for nuclei where the $(s,d,g)$- and $(p,f,h)$-orbitals are the valence orbitals, not an $\hbar\omega$ appropriate for $^{12}$C. We choose $\hbar\omega = 7.2$ MeV. The constant in Eq. (6) is again chosen so that when the formula is used for the $0p_{1/2}$ orbital it reproduces the experimental value, $-4.95$ MeV. The values of the single-particle energies used are given in Table II.

## IV. FORMALISM

We apply the results of shell-model calculations just described to the evaluation of a number of isovector weak and electromagnetic transitions in the $A = 12$ system. We consider the inclusive neutrino reactions

$$\nu_\mu + {}^{12}C \rightarrow X + \mu^-, \tag{8}$$

$$\nu_e + {}^{12}C \rightarrow X + e^-, \tag{9}$$

where $X$ is unobserved, the muon capture process

$$\mu^-(1S) + {}^{12}C \rightarrow X + \nu_\mu, \tag{10}$$

and photoabsorption

$$\gamma + {}^{12}C \rightarrow X, \tag{11}$$

which proceeds only through the isovector E1 multipole. In addition, we take guidance from earlier calculations [13,14] for the $(p,n)$, $(n,p)$ and $(e,e')$ reactions to the isovector giant resonances of interest. Further we consider the exclusive cross section to the ground state of $^{12}$N for the first three of the above reactions, as well as the Gamow-Teller beta decay from $^{12}$N to the ground state of $^{12}$C.

The formalism for calculating these cross sections has been given by O'Connell, Donnelly and Walecka [15]. For the neutrino absorption reactions, Eqs. (8) and (9), the cross section is given by

$$\sigma(E_\nu) = \frac{G^2}{2\pi} \sum_f \int_{-1}^{+1} d(\cos\theta) \delta(E_i - E_f + E_\nu - E_e) p_\ell E_\ell \mid M \mid^2, \tag{12}$$

where $G$ is the weak interaction coupling constant, $G/(\hbar c)^3 = 1.16639 \times 10^{-5}$ GeV$^{-2}$, $p_\nu, E_\nu$ are the neutrino momentum and energy in the lab system, $p_\ell, E_\ell$ are the outgoing lepton momentum and energy, $\cos\theta$ is the cosine of the angle between the electron and neutrino directions, and $E_i - E_f$ is the mass difference between the initial and final nuclei. It is convenient to define an energy and momentum transfer: $q_0 = E_i - E_f = E_\ell - E_\nu$ and $\mathbf{q} = \mathbf{p}_\ell - \mathbf{p}_\nu$. There is a sum over all final nuclear states, which in our calculations becomes a sum over all the states available in a given shell-model calculation.

Lastly $\mid M \mid^2$ is given schematically

$$\mid M \mid^2 = \sum_{\lambda=0}^{\infty} (L_1 W_1^{(\lambda)} + L_2 W_2^{(\lambda)} + L_3 W_3^{(\lambda)} + L_4 W_4^{(\lambda)} + L_5 W_5^{(\lambda)}) \tag{13}$$

where $L_1 \ldots L_5$ are the five lepton traces given in Table II of Ref. [15] and are functions of $q_0$, $\mathbf{q}$, and $\cos\theta$, while $W_1^{(\lambda)} \ldots W_5^{(\lambda)}$ are certain combinations of squares of nuclear matrix elements. The sum is over all multipolarities of operators, $\lambda$, satisfying angular momentum condition $\Delta(J_i, J_f, \lambda)$, where $J_i$ and $J_f$ are the initial and final spins.

In general, there are seven transition operators, $\mathcal{O}^{(\lambda)}(q\mathbf{x})$, detailed in the tables of Donnelly and Haxton [16]. The operators are functions of $q$, the magnitude of the momentum transfer, $q = \mid \mathbf{q} \mid$, and $\mathbf{x}$, the position coordinate. Reduced matrix elements of these operators can be decomposed into two factors:

$$\langle J_f \parallel \mathcal{O}^{(\lambda)}(q\mathbf{x}) \parallel J_i \rangle = \sum_{j_\alpha, j_\beta} \langle J_f \parallel [a^\dagger_{j_\alpha}, a_{j_\beta}]^{(\lambda)} \parallel J_i \rangle \langle j_\alpha \parallel \mathcal{O}^{(\lambda)}(q\mathbf{x}) \parallel j_\beta \rangle, \tag{14}$$

where the accompanying isospin quantum numbers have been suppressed for economy of notation. The first factor is the expectation value of shell-model creation and annihilation operators evaluated with the many-body shell-model wave functions and is known as the one-body density matrix element (obdme). The second factor, the single-particle matrix element (spme), is independent of the many-body wave functions but depends on the transition process in question through the momentum transfer, $q$. This factorization enables the shell-model calculations to be separated from the transition rate calculations. Files of obdme prepared in the former are transferred to the latter. The spme also involve radial integrals of spherical Bessel functions, $j_L(qx)$, or their derivatives with two single-particle nuclear radial functions. For the latter, we use Woods-Saxon functions. The nuclear matrix elements are also multiplied by form factors, which are functions of $q$, as given in Ref. [15].

In the shell-model calculations, the center-of-mass is treated on an equal footing with all other $3(A-1)$ coordinates, and which causes an erroneous center-of-mass contribution to the neutrino cross sections and muon capture rate. To correct for this we multiply the computed cross section by the Tassie-Barker function $\mid g_{\rm cm}(q) \mid^2$, where

$$g_{\rm cm}(q) = e^{y/A} \tag{15}$$

with $y = (bq/2)^2$, $q$ the momentum transfer, $b$ the oscillator length parameter and $A$ the nuclear mass number. For $y \sim 1$ and $A = 12$, this correction provides a 20% increase in the computed cross section.[1]

In the expression for the neutrino absorption cross-section, Eq. (12), one further correction has to be applied that represents the distortion of the outgoing lepton in the Coulomb field of the daughter nucleus. We are guided by the work of Engel [17] in this matter. The energy of the emerging lepton, while under the influence of a constant electrostatic potential within the nucleus, is effectively shifted:

$$\begin{aligned} E_{\ell,\rm eff} &= E_\ell - V(0) \\ p_{\ell,\rm eff} &= \left(E^2_{\ell,\rm eff} - m^2\right)^{1/2} \\ V(0) &= -\frac{3}{2}\frac{Z\alpha}{R} \end{aligned} \tag{16}$$

where $E_\ell$, $p_\ell$ and $m$ are the energy, momentum and mass of the emerging lepton, $Z$ the charge number for the daughter nucleus (with a plus sign for particle leptons, and a minus sign for antiparticle leptons), $R$ the nuclear radius and $\alpha$ the fine-structure constant. If $p_\ell R$ is less than $\sim 0.5$, then the lepton wave function is primarily $s$-state, and the usual Fermi function, $F(Z, E_\ell)$ is used as a multiplicative correction factor in Eq. (12). For DAR cross sections, $p_\ell R$ is always less than 0.5. For DIF cross sections, the emerging muons have $p_\ell R > 0.5$ for most of the energy range. Then we follow Engel's suggestion [17] that the cross-section be multiplied by a factor $p_{\ell,\rm eff}E_{\ell,\rm eff}/(p_\ell E_\ell)$ and the momentum transfer $q = |\mathbf{q}|$ used in the spherical Bessel functions in the radial integrals be shortened by using

[1]This correction has been derived with the use of oscillator functions. We apply the same correction even when using Woods-Saxon radial functions.

$\mathbf{q}_{\rm eff} = \mathbf{p}_{\ell,\rm eff} - \mathbf{p}_\nu$. The multiplicative factor increases the DIF cross section by about 16% while the shortening of $q$ in the Bessel functions reduces this enhancement to about 11%.

In the results to be given in the next section, for the neutrino cross sections Eqs. (8) and (9), we will give the flux-averaged cross-sections defined as

$$\overline{\sigma} = \int dE_\nu \Lambda(E_\nu)\sigma(E_\nu) / \int dE_\nu \Lambda(E_\nu), \tag{17}$$

where $\Lambda(E_\nu)$ is the incident neutrino flux. For the liquid scintillator neutrino detector (LSND) experiment [18], we use their most recently determined muon-neutrino flux [19] from pion decays in flight. For the experiments by the KARMEN Collaboration [20], the electron-neutrino beam comes from muon decays at rest and the flux is given by the Michel spectrum.

The muon capture rate, Eq. (10), is given by [15]

$$\Lambda_c = \frac{G^2 E_\nu^2}{2\pi} \mid \phi_{1S} \mid^2_{\rm av} \left(1 + \frac{E_\nu}{m_T}\right)^{-1} \mid M \mid^2, \tag{18}$$

where the outgoing neutrino energy is $E_\nu = m_\ell - \epsilon_b + E_i - E_f$, with $m_\ell$ the muon reduced mass, $\epsilon_b$ the muon K-shell binding energy and $E_i - E_f = q_0$ is the mass difference between the initial and final nuclear states. The factor $(1 + E_\nu/m_T)^{-1}$ is a recoil correction, with $m_T$ the target nucleus mass. Further, $\phi_{1S}$ is the muon K-shell bound state wave function evaluated at the nucleus

$$\mid \phi_{1S} \mid^2_{\rm av} = \frac{R}{\pi}(Z\alpha m_\ell)^3 \tag{19}$$

with $R$ a reduction factor for the finite size of the nuclear charge distribution, $R = 0.86$ for $^{12}$C. The total inclusive capture rate is the sum of Eq. (18), summed over all final states contained in the shell-model calculation.

Finally, the photoabsorption cross section, Eq. (11), is given by [12]

$$\sigma(E_\gamma) = \frac{16\pi^3}{9} \alpha E_\gamma B(E1; i \to f) \delta(E_\gamma - E_f + E_i) \tag{20}$$

where $E_\gamma$ is the photon energy and $B(E1; i \to f)$ the reduced transition probability for $E1$ photon absorption. Again the total absorption cross section, $\sigma_{\rm tot}$, is given by the sum of Eq. (20), summed over all final states contained in the shell-model calculation. In the different calculations, the total absorption cross section reaches a saturation value at a photon energy $E_\gamma$ of the order of 50 MeV, slightly less in $1\hbar\omega$ calculations, slightly more in $(1+3)\hbar\omega$. Note, only the $E1$ multipole is retained in the calculations. The experimental total absorption cross section [27], however, shows no sign of saturating and keeps growing with increasing $E_\gamma$. This is because higher multipoles are contributing. In quoting an experimental value, we have cut off the contributions at $E_\gamma = 50 \pm 5$ MeV for a total cross section of $\sigma_{\rm tot} = 21 \pm 1$ fm$^2$.

## V. RESULTS

### A. Inclusive Cross-sections

We present the results of our calculations in Tables III to VI. In most of our tables the tabulated results are for Woods-Saxon radial wave functions. In Table IV we list the results for three sets of radial wave functions, Woods-Saxon, and harmonic oscillators with $b = 1.82$ fm and $b = 1.64$ fm. For each of the four model spaces, we give the results as a function of the size of the particle-hole space, increasing from $0\hbar\omega$ excitations up to $4\hbar\omega$. In Table III are the inclusive cross sections for $(\nu_\mu, \mu^-)$-scattering, $(\nu_e, e^-)$-scattering, muon capture, and photoabsorption on $^{12}$C. The calculation in the TDA approximation overestimates the cross sections in all cases, while the two variants of RPA reduce these cross sections by about a factor of two and are in closer agreement with experiment. For the $(\nu_\mu, \mu^-)$ cross section, the calculations to $4\hbar\omega$ in the closed-shell RPA, closed-shell RPA + 2p-2h, and unrestricted shell model yield 18.2, 16.7 and $13.8 \times 10^{-40}$ cm$^2$, the latter value being an extrapolation from a $2\hbar\omega$ calculation. This latter value, also, is within range of the experimental value of $(12.4 \pm 1.8) \times 10^{-40}$ cm$^2$ [18]. This is our principal result. In ref [1], with the $p_{3/2}$ subshell taken as a closed shell, Kolbe *et al.* obtain a result $19.8(5) \times 10^{-40}$ cm$^2$, where the uncertainty represents a spread between different choices of the effective interaction. Recently [2], this work has been extended to include a partial occupation of the $p_{1/2}$ subshell obtaining a result some two units smaller of $17.6(2) \times 10^{-40}$ cm$^2$. The present calculation reduces this four units further with the use of fully unrestricted shell-model configurations and Woods-Saxon radial wave functions.

However, in obtaining an improved result for the DIF cross section, we must check that the same calculation still satisfactorily reproduces other relevant data. In Table III we therefore give results for $(\nu_e, e^-)$ DAR cross sections, $\mu$-capture rates and photoabsorption cross sections. For $(\nu_e, e^-)$ neutrino absorption, we calculate in closed-shell RPA, closed-shell RPA + 2p-2h, and unrestricted shell-model cross sections of 21.9, 20.4 and 12.5 $\times 10^{-42}$ cm$^2$ respectively compared with the KARMEN result of $(14.0 \pm 1.2) \times 10^{-42}$ cm$^2$ and the LSND result of $(14.8 \pm 1.3) \times 10^{-42}$ cm$^2$. Again the unrestricted shell-model calculation gives a big reduction to the neutrino absorption cross sections resulting, in this case, with a value that is on the low side compared to experiment. However, it is only outside the experimental range by one standard deviation. By comparison, the continuum RPA calculation of Kolbe *et al.* [2] obtains a result $14.4 \times 10^{-42}$ cm$^2$ exactly in the right range. Thus it would seem that in obtaining a smaller DIF cross section as required, we are simultaneously underpredicting the DAR cross section.

For $\mu$-capture and photoabsorption, our results in Table III indicate the unrestricted shell-model calculation slightly overpredicting the experimental result, but only by about 10%. In summary, then, our improved result for the DIF cross section comes with some modest deterioration in the DAR, $\mu$-capture and photoabsorption results.

### B. The Radial Wave Functions

In discrete-state shell-model calculations with effective interactions, the choice of radial wave functions to use in evaluating transition matrix elements remains unspecified. There is therefore some freedom in making this choice. In Table IV we give results for three choices

of radial functions for the inclusive cross sections, but in this case explicitly exclude the ground-state contribution.

A description of the rms charge radius of $^{12}$C using harmonic oscillator single-particle wave functions requires an oscillator parameter of $b = 1.64$ fm. However, to obtain reasonably good fits to the shape of the electron-scattering form factors in $^{12}$C considerably different values of $b$ are often needed. An analysis [21,13,14] of the $(e, e')$ form factors and $(p, p')$ inelastic scattering data suggests the need for state dependent oscillator parameters in the broad range $1.64 - 1.94$ fm. The problem arises as a result of the inability to describe loosely bound or unbound states of $^{12}$N using the same oscillator parameter as is needed to describe the deeply bound nucleons in the $^{12}$C ground state.

In a systematic study of the effect of more realistic single-particle wave function on the $^{12}$C$(p, n)$ reaction Millener *et al.* [13,14] concluded that the use of Woods-Saxon wave functions reduces the predicted cross sections by about 30% over the harmonic oscillator predictions. Ohnuma *et al.* [22] drew a similar conclusion. The large suppression of the cross section with the use of Woods-Saxon single particle wave functions is because of the large mismatch between the radial wave functions describing the $^{12}C_{g.s.}$ versus the $^{12}$N states. An effect of similar magnitude was found for the $^{12}$C$(e, e')$ form factors.

The cross sections obtained using Woods-Saxon wave functions have been found [21,23] to be quite similar to those obtained when the harmonic oscillator parameter is chosen to fit best the $(e, e')$ form factor. Brady *et al.* [13] obtained a good fit to the $(e, e')$ form factor for the $^{12}C_{g.s.} \rightarrow ^{12}$C(15.11 MeV) using an oscillator parameter $b = 1.82$ fm. To examine the sensitivity of the predicted neutrino cross sections to the assumed oscillator parameter we recalculated the cross sections using a smaller value of $b$, and the results are tabulated in Table IV. Relative to the $b = 1.82$ fm calculation the inclusive $(\nu_\mu, \mu^-)$ DIF cross section increased only slightly for $b = 1.64$ fm, while the $(\nu_e, e^-)$ DAR cross section decreased by 17% and the $\mu$-capture rate decreased by 14%. The larger value of $b$ in all cases gave predictions closer in agreement with experiment.

The difference in the response of the three reactions to the larger oscillator parameter reflects the different average momentum transfer involved in each, DIF ($q \sim 1$ fm$^{-1}$), DAR ($q \sim 0.2$ fm$^{-1}$), and $\mu$-capture ($q \sim 0.5$ fm$^{-1}$). Changing the oscillator parameter changes the shape of the axial-vector and vector form factors entering the neutrino cross sections. For momentum transfers below the first maximum of the form factor the neutrino cross section increases as $b$ increases. In contrast, momentum transfers beyond the first peak in the form factor result in the predicted cross section being decreased as $b$ increases.

We next consider the use of Woods-Saxon radial wave functions. The mismatch between the initial and final radial wave function, resulting from binding energy changes, cannot be accounted for within a harmonic oscillator basis. It is impossible to treat the binding energy of the final nucleon rigorously within a discrete shell model, but an estimate of the effect of binding energies on the predicted cross sections can be obtained using Woods-Saxon wave functions that are only just bound. The Woods-Saxon result reduces the predicted inclusive DIF $(\nu_\mu, \mu^-)$ cross section by about six units, putting the result within the experimental range. However the predicted DAR $(\nu_e, e^-)$ cross-section is similarly reduced by 2.8 units and is too small compared with experiment. The calculated $\mu$-capture rate increases and slightly overpredicts experiment.

The use of Woods-Saxon functions causes some radial integrals to become non-zero,

where they are identically zero for oscillator wave functions. This is seen easily for the E1 $\gamma$-ray transition matrix elements. With oscillators, the predicted B(E1) strength in $(0+1+2+3)\hbar\omega$ is slightly less than in a $(0+1)\hbar\omega$ calculation. But with the use of Woods-Saxon wave functions the $(0+1+2+3)\hbar\omega$ calculation is about 40% larger. This is due to the non-vanishing integral $\int R(n,l)rR(n',l')r^2dr$, when radial functions, $R(n,l)$ and $R(n',l')$, represent single-particle states differing in energy by $3\hbar\omega$. The same effect is noticeable in the calculated $\mu$-capture rate and to some extent in neutrino scattering cross sections.

### C. The Exclusive Cross Section to the $^{12}\mathrm{N}_{g.s.}$

Next, we examine in detail the exclusive cross section to the $^{12}$N ground state. The results are in Table V. The first row of the TDA calculation labeled $0\hbar\omega$ is the single-particle $0^+ \rightarrow (p_{3/2}^{-1}, p_{1/2})1^+$ transition rate, while the first row of the unrestricted shell-model calculations represents the complete $p$-shell mixing as given by Cohen-Kurath (8-16)2BME wave functions. We note that the cross sections in the configuration-mixed calculations are a factor four to five smaller than the single-particle estimate. The closed-shell RPA variants are only able to obtain about a factor of two of these reductions. Increasing the particle-hole space to include $2\hbar\omega$ and $4\hbar\omega$ excitations only has an impact in the $10-20\%$ range for neutrino absorption, muon capture and beta decay.

In our complete unrestricted $(0+2)\hbar\omega$ calculations we paid particular attention to handling the strong $\Delta\hbar\omega = 2$ interaction in a consistent way. When empirical or realistic Hamiltonians which give a reasonable description of $0\hbar\omega$ and $1\hbar\omega$ spectra are used in full $(0{+}2)\hbar\omega$ spaces, the $\Delta\hbar\omega = 2$ interaction pushes the ground state down by several MeV. The resulting spectrum is in poor agreement with experiment. With no restrictions placed on our $(0{+}1{+}2)\hbar\omega$ calculations we found that the giant monopole resonance was predicted to lie too low in the spectrum and large percentages of $1p1h$ excitations were predicted in the ground state wave function. To correct for this pathological behavior we adjusted diagonal matrix elements of the Hamiltonian so as to restore the energy of the giant monopole resonance to lie above 20 MeV, as expected by self-consistent RPA calculation using density-dependent interactions. In addition we set the strong $\langle 1p1h\ 2\hbar\omega|H|0\hbar\omega\rangle$ matrix elements, that transform as $(\lambda,\mu) = (2,0)$ under SU(3), to zero. The resulting $^{12}$C ground-state wave function yields a predicted strength for the GT transition to the $^{12}\mathrm{N}_{g.s.}$ that is in reasonable agreement with experiment, Table V.

### D. The Multipole Decomposition

Finally, in Table VI we give the breakdown in the contribution to the cross sections from each multipole, for oscillator wave functions using $b = 1.82$ fm. The most striking result is that 80% of the $(\nu_e, e^-)$ inclusive cross sections comes from the ground-state $1^+$ multipole, with the remaining 20% coming from the excited state $1^+$, $1^-$, and $2^-$ multipoles. Muon capture also is dominated by the ground-state $1^+$ multipole, being, in this case, 65% of the inclusive rate. The remaining 35% is dominated by the $1^-$ multipole with lesser contributions from the $2^-$, $0^-$, excited $1^+$, and $2^+$ multipoles. By contrast, the $(\nu_\mu, \mu^-)$

inclusive cross section gets only a 10% contribution from the ground-state $1^+$ multipole, while the remaining 90% is spread over many multipoles, $1^-$ to $4^-$ and $1^+$ to $3^+$ dominating.

## VI. CONCLUSIONS

We have examined the $(\nu_\mu, \mu^-)$, $(\nu_e, e^-)$, and $\mu$-capture reactions on $^{12}$C within a shell-model calculations that included up to $4\hbar\omega$ of excitation. Calculations were carried out for four separate model spaces, for three sets of radial wave functions. Closest agreement between theory and experiment was obtained for the most sophisticated model spaces, which included both RPA-type correlations and an open $p$-shell $^{12}$C ground state.

Previous analyses of the $(e, e')$, $(p, n)$ and $(n, p)$ reactions have shown that when the large difference in binding energies between the $^{12}$C ground state and the $^{12}$N final states in taken into account, the predicted cross sections are reduced by about 30% relative to harmonic oscillator shell-model results. To examine the effect of more realistic single-particle wave functions we compared the predictions using two oscillator parameters, $b = 1.64$ fm and $b = 1.82$ fm and Woods-Saxon wave functions. The lower value of $b$ is suggested by the $^{12}$C ground-state charge radius, while the larger value comes from a fit to the shape of the transverse form factor for the $^{12}\mathrm{C}(0 \rightarrow 15.11; 1^+)$ transition. Increasing $b$ increases the predicted cross section for all low momentum transfer reactions. The photoabsorption cross section increases with $b^2$, and the DAR $(\nu_e, e^-)$ cross-section increased by a smaller but similar amount. For higher momentum transfer processes ($q \sim 1$ fm$^{-1}$), such as the DIF neutrino reaction and the $(p, n)$ reaction at intermediate energies, the predicted cross section decreases with increasing $b$. Of course, binding energy differences result in a mismatch between the initial and final radial wave functions which cannot be mimicked by a change in the oscillator parameter, and fits to the $(e, e')$ form factors for states in $^{12}$C within an oscillator basis find a need for state-dependent oscillator parameters in the range $1.64 - 1.94$ fm. Incorporating binding energy effects through the use of Woods-Saxon wave function has a large effect on the predicted cross sections for all reactions. The predicted inclusive DIF $(\nu_\mu, \mu^-)$ cross section is in agreement with the experimental range. However, the Woods-Saxon calculation underpredicts the DAR $(\nu_e, e^-)$ cross section and overpredicts the $\mu$-capture rate.

Given the model dependence noted in the present set of calculations, it is difficult to determine the expected DIF cross section to high accuracy using constraints from other observables. For example, the ratio of the predicted DAR to DIF cross sections changed by about 20% for the two different values of oscillator parameter considered. Furthermore, we were unable to include a fully realistic treatment of the continuum nature of states of $^{12}$N. However the success of the Woods-Saxon calculation for the DIF cross section while only failing to obtain the experimental result for the DAR cross section, $\mu$-capture rate and photoabsorption cross section by one to two standard deviations indicates the importance of a continuum treatment for the final states, coupled with an open-shell description of the nuclear configurations.

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## TABLES

TABLE I. Spin structure of the $^{12}$C ground state

| Model | S = 0 | S = 1 | S = 2 | $\%[\widetilde{444}]$ |
|---|---|---|---|---|
| Three $\alpha-$particles | 100% | 0 | 0 | 100% |
| Cohen-Kurath Interaction | 81% | 18% | $< 2\%$ | 78% |
| $(p_{3/2})^8$ | 16% | 59% | 25% | 6% |

TABLE II. Single-particle energies.

| $0s_{1/2}$ | $0p_{3/2}$ | $0p_{1/2}$ | $0d_{5/2}$ | $1s_{1/2}$ | $0d_{3/2}$ | $0f_{7/2}$ | $1p_{3/2}$ | $0f_{5/2}$ | $1p_{1/2}$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| −34.40 | −12.26 | −4.95 | 5.98 | 3.70 | 8.97 | 10.16 | 8.78 | 9.39 | 8.39 | |
| $0g_{9/2}$ | $1d_{5/2}$ | $2s_{1/2}$ | $0g_{7/2}$ | $1d_{3/2}$ | $0h_{11/2}$ | $1f_{7/2}$ | $2p_{3/2}$ | $0h_{9/2}$ | $1f_{5/2}$ | $2p_{1/2}$ |
| 13.87 | 18.09 | 20.46 | 18.28 | 20.54 | 20.17 | 25.32 | 28.61 | 25.56 | 28.75 | 30.08 |

TABLE III. Inclusive cross sections involving a continuum of $^{12}$N states, including the ground state. Woods-Saxon radial functions are used, with their asymptotic forms matched to the experimental separation energies for bound states, and matched to a binding energy of 0.01 MeV for unbound states.

| | $(\nu_\mu, \mu^-)$ DIF $\overline{\sigma} \times 10^{-40}$ cm$^2$ | $(\nu_e, e^-)$ DAR $\overline{\sigma} \times 10^{-42}$ cm$^2$ | $\mu$-capture $\Lambda_c \times 10^3$ s$^{-1}$ | photoabsorption $\sigma_{\rm tot} \times 10^{-26}$ cm$^2$ |
|---|---|---|---|---|
| Closed-shell TDA | | | | |
| $0\hbar\omega$ | 3.8 | 34.6 | 31.0 | |
| $(0+1)\hbar\omega$ | 12.8 | 39.0 | 62.9 | 22.7 |
| $(0+1+2)\hbar\omega$ | 26.1 | 40.0 | 67.7 | |
| $(0+1+2+3)\hbar\omega$ | 31.0 | 40.7 | 74.0 | 29.5 |
| $(0+1+2+3+4)\hbar\omega$ | 33.7 | 40.6 | 75.9 | |
| Closed-shell RPA | | | | |
| $0\hbar\omega$ | 1.7 | 13.3 | 11.7 | |
| $(0+1)\hbar\omega$ | 6.9 | 15.8 | 35.1 | 14.4 |
| $(0+1+2)\hbar\omega$ | 13.2 | 20.9 | 41.4 | |
| $(0+1+2+3)\hbar\omega$ | 16.2 | 21.4 | 43.5 | 18.3 |
| $(0+1+2+3+4)\hbar\omega$ | 18.2 | 21.9 | 45.4 | |
| Closed-shell RPA + 2p-2h | | | | |
| $0\hbar\omega$ | 1.9 | 14.6 | 13.4 | |
| $(0+1)\hbar\omega$ | 7.1 | 17.0 | 36.2 | 17.9 |
| $(0+1+2)\hbar\omega$ | 13.6 | 21.1 | 42.5 | |
| $(0+1+2+3)\hbar\omega$ | 16.1 | 21.3 | 43.5 | 21.6 |
| $(0+1+2+3+4)\hbar\omega$ | 16.7 | 20.4 | 44.1 | |
| Unrestricted shell model | | | | |
| $0\hbar\omega$ | 1.3 | 7.1 | 6.4 | |
| $(0+1)\hbar\omega$ | 7.6 | 10.5 | 31.6 | 19.6 |
| $(0+1+2)\hbar\omega$ | 11.1 | 12.1 | 39.5 | |
| $(0+1+2+3)\hbar\omega$ | (13.2)[a] | (12.3)[a] | (40.6)[a] | (23.6)[a] |
| $(0+1+2+3+4)\hbar\omega$ | (13.8)[a] | (12.5)[a] | (42.2)[a] | |
| CRPA [2] | 17.6(2) | 14.4(1) | 38.0(7) | |
| Expt. | 12.4(18) [18] | 14.0(12) [20]<br>14.8(13) [24,18] | 38.9(9) [25,26] | 21(1) [27] |

[a]Estimate

TABLE IV. Inclusive cross sections involving a continuum of $^{12}$N states, but excluding the ground state, for various choices of radial wavefunctions: oscillator functions with length parameter $b = 1.64$ fm or 1.82 fm, and Woods-Saxon functions.

| | $(\nu_\mu, \mu^-)$ DIF $\overline{\sigma} \times 10^{-40}$ cm$^2$ | $(\nu_e, e^-)$ DAR $\overline{\sigma} \times 10^{-42}$ cm$^2$ | $\mu$-capture $\Lambda_c \times 10^3$ s$^{-1}$ | photoabsorption $\sigma_{\rm tot} \times 10^{-26}$ cm$^2$ |
|---|---|---|---|---|
| Oscillator functions ($b = 1.64$ fm) | | | | |
| Closed-shell RPA + 2p-2h | | | | |
| $0\hbar\omega$ | 1.5 | 1.1 | 1.5 | |
| $(0+1)\hbar\omega$ | 9.4 | 4.9 | 22.6 | 17.9 |
| $(0+1+2)\hbar\omega$ | 14.2 | 5.3 | 25.9 | |
| $(0+1+2+3)\hbar\omega$ | 16.5 | 5.1 | 24.6 | 16.8 |
| $(0+1+2+3+4)\hbar\omega$ | 17.1 | 5.5 | 25.0 | |
| Unrestricted shell model | | | | |
| $0\hbar\omega$ | 1.3 | 0.3 | 0.7 | |
| $(0+1)\hbar\omega$ | 11.0 | 5.6 | 27.1 | 22.3 |
| $(0+1+2)\hbar\omega$ | 15.9 | 5.7 | 28.9 | |
| $(0+1+2+3)\hbar\omega$ | (18.5)[a] | ( 5.5)[a] | (27.5)[a] | (20.9)[a] |
| $(0+1+2+3+4)\hbar\omega$ | (19.1)[a] | ( 5.9)[a] | (27.9)[a] | |
| Oscillator functions ($b = 1.82$ fm) | | | | |
| Closed-shell RPA + 2p-2h | | | | |
| $0\hbar\omega$ | 1.6 | 1.1 | 1.6 | |
| $(0+1)\hbar\omega$ | 8.9 | 5.6 | 25.5 | 22.1 |
| $(0+1+2)\hbar\omega$ | 13.6 | 5.9 | 29.5 | |
| $(0+1+2+3)\hbar\omega$ | 16.3 | 5.7 | 27.9 | 20.7 |
| $(0+1+2+3+4)\hbar\omega$ | 17.2 | 6.1 | 28.3 | |
| Unrestricted shell model | | | | |
| $0\hbar\omega$ | 1.3 | 0.3 | 0.9 | |
| $(0+1)\hbar\omega$ | 10.3 | 6.6 | 30.8 | 27.5 |
| $(0+1+2)\hbar\omega$ | 15.1 | 6.6 | 33.0 | |
| $(0+1+2+3)\hbar\omega$ | (18.1)[a] | ( 6.4)[a] | (31.2)[a] | (25.7)[a] |
| $(0+1+2+3+4)\hbar\omega$ | (19.1)[a] | ( 6.9)[a] | (31.7)[a] | |
| Woods-Saxon functions | | | | |
| Closed-shell RPA + 2p-2h | | | | |
| $0\hbar\omega$ | 0.8 | 0.6 | 1.3 | |
| $(0+1)\hbar\omega$ | 6.0 | 3.0 | 24.1 | 17.9 |
| $(0+1+2)\hbar\omega$ | 12.4 | 3.1 | 27.7 | |
| $(0+1+2+3)\hbar\omega$ | 14.9 | 3.3 | 28.6 | 21.6 |
| $(0+1+2+3+4)\hbar\omega$ | 15.6 | 3.5 | 29.9 | |
| Unrestricted shell-model | | | | |
| $0\hbar\omega$ | 0.7 | 0.2 | 0.8 | |

| | | | | |
|---|---|---|---|---|
| $(0+1)\hbar\omega$ | 7.0 | 3.6 | 26.0 | 19.6 |
| $(0+1+2)\hbar\omega$ | 10.5 | 3.7 | 32.9 | |
| $(0+1+2+3)\hbar\omega$ | (12.6)[a] | ( 3.8)[a] | (34.0)[a] | (23.6)[a] |
| $(0+1+2+3+4)\hbar\omega$ | (13.2)[a] | ( 4.1)[a] | (35.6)[a] | |
| CRPA [2] | 16.9(2) | 5.5(1) | 32.0(7) | |
| Expt. | 11.7(18) [18] | 5.1(8) [20]<br>5.7(8) [24] | 32.8(8) [25] | 21(1) [27] |

[a]Estimate

TABLE V. Exclusive cross sections involving the ground state of $^{12}$N only. Woods-Saxon radial functions are used, whose asymptotic forms are matched to the experimental ground-state separation energies.

| | $(\nu_\mu, \mu^-)$ DIF<br>$\overline{\sigma} \times 10^{-40}$ cm$^2$ | $(\nu_e, e^-)$ DAR<br>$\overline{\sigma} \times 10^{-42}$ cm$^2$ | $\mu$-capture<br>$\Lambda_c \times 10^3$ s$^{-1}$ | $\beta$-decay<br>$B(GT; 1^+ \to 0^+)$ |
|---|---|---|---|---|
| Closed-shell TDA | | | | |
| $0\hbar\omega$ | 2.74 | 34.5 | 30.0 | 2.26 |
| $(0+1+2)\hbar\omega$ | 2.41 | 35.4 | 30.7 | 2.45 |
| $(0+1+2+3+4)\hbar\omega$ | 2.42 | 35.2 | 30.6 | 2.44 |
| Closed-shell RPA | | | | |
| $0\hbar\omega$ | 1.07 | 13.2 | 11.1 | 0.87 |
| $(0+1+2)\hbar\omega$ | 1.19 | 18.2 | 15.1 | 1.26 |
| $(0+1+2+3+4)\hbar\omega$ | 1.24 | 18.8 | 15.6 | 1.30 |
| Closed-shell RPA + 2p-2h | | | | |
| $0\hbar\omega$ | 1.17 | 14.0 | 12.1 | 0.91 |
| $(0+1+2)\hbar\omega$ | 1.16 | 18.0 | 14.9 | 1.23 |
| $(0+1+2+3+4)\hbar\omega$ | 1.12 | 17.0 | 14.2 | 1.16 |
| Unrestricted shell model | | | | |
| $0\hbar\omega$ | 0.59 | 6.9 | 5.7 | 0.45 |
| $(0+1+2)\hbar\omega$ | 0.58 | 8.4 | 6.6 | 0.56 |
| $(0+1+2+3+4)\hbar\omega$ | ( 0.56)[a] | ( 7.9)[a] | ( 6.3)[a] | (0.53)[a] |
| CRPA [2] | 0.7(3) | 8.9(1) | 6.0(3) | |
| Expt. | 0.66(14) [18] | 8.9(10) [20]<br>9.1(10) [18] | 6.1(3) [26] | 0.50(3) [28] |

[a]Estimate

TABLE VI. Contribution from each multipole to the inclusive cross section in a closed-shell RPA + 2p-2h model space. The values on excluding the $^{12}$N ground-state contribution are given in brackets.

| | $(\nu_\mu, \mu^-)$ DIF $\overline{\sigma} \times 10^{-40}$ cm$^2$ | $(\nu_e, e^-)$ DAR $\overline{\sigma} \times 10^{-42}$ cm$^2$ | $\mu$-capture $\Lambda_c \times 10^3$ s$^{-1}$ |
|---|---|---|---|
| Positive parity | | | |
| $0^+$ | 0.11 | 0.00 | 0.21 |
| $1^+$ | 2.95 ( 1.79) | 22.52 (1.66) | 15.43 ( 3.87) |
| $2^+$ | 2.59 | 0.08 | 1.36 |
| $3^+$ | 1.39 | 0.03 | 0.46 |
| $4^+$ | 0.66 | 0.00 | 0.00 |
| $5^+$ | 0.51 | 0.00 | 0.00 |
| $6^+$ | 0.04 | 0.00 | 0.00 |
| $\geq 7^+$ | 0.04 | 0.00 | 0.00 |
| Sum | 8.29 ( 7.13) | 22.63 (1.77) | 17.48 ( 5.92) |
| Negative parity | | | |
| $0^-$ | 0.07 | 0.04 | 2.12 |
| $1^-$ | 3.55 | 1.90 | 12.25 |
| $2^-$ | 2.91 | 2.36 | 7.79 |
| $3^-$ | 1.77 | 0.00 | 0.11 |
| $4^-$ | 1.41 | 0.00 | 0.07 |
| $5^-$ | 0.17 | 0.00 | 0.00 |
| $6^-$ | 0.17 | 0.00 | 0.00 |
| $\geq 7^-$ | 0.00 | 0.00 | 0.00 |
| Sum | 10.04 | 4.30 | 22.34 |
| Total | 18.33 (17.17) | 26.93 (6.07) | 39.82 (28.26) |